

**Sven Utcke**

# Transfer and Invariants of Surfaces of Revolution

**Diploma Thesis**

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DIPLOMARBEIT

Sven Utcke

# Transfer and Invariants of Surfaces of Revolution

Erstellt in Zusammenarbeit mit



UNIVERSITY OF OXFORD  
Department of Engineering Science

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## Transfer and Invariants of Surfaces of Revolution

### 0.1 Introduction

A number of recent papers in the Computer Vision and Pattern Recognition literature have demonstrated that invariants, or equivalently structure modulo a 3D linear transformation, are sufficient for object recognition [1, 19, 20]. The final stage in the recognition process is verification, where an outline is *transferred* from an acquisition image of the object to the target image.

For the most part recognition based on invariants has concentrated on planar objects [19], though recently 3D invariants have been measured from single and multiple images for polyhedra [18], point sets [9, 13, 18], surfaces of revolution [12] and algebraic surfaces [11]. The work so far on surfaces of revolution has only exploited isolated points on the outline (such as bitangents), and has not addressed transfer or verification.

The aim of this project is to extend the transfer and extraction of invariants to surfaces of revolution using the entire outline.

### 0.2 Specification

A surface of revolution is simply a rotated *generating curve*. There are two related goals for the project:

1. **Transfer:** Given a single (or multiple) views of the surface, obtain the projection in any other given view. For example, after specifying a

minimal number of points in the target image, render the object from that viewpoint.

2. **Invariants:** Extract from the outline in a single view a *signature* or set of invariants which are viewpoint independent. These should also be derived directly from the generating curve.

The project will be developed in a number of stages. In the first place the affine approximation to projection will be employed (so that the object is imaged under parallel projection). This has the virtue that the essential geometry of the contour generator depends only on one parameter - the direction of projection. The second stage will investigate perspective projection, where (two) parameters specifying the optical center must be considered.

The analysis will be partly theoretical - employing the symbolic algebra package Mathematica, and partly experimental. The goal is to develop methods that work reliably and robustly on images of real objects.

### 0.3 Relevant Literature

Papers listed below on the application of invariants to model based recognition. Also background texts on projective geometry [16, 22, 24] and differential geometry [8, 15, 21].

<i>Referent:</i>	Prof. Dr.-Ing. H. Burkhardt
<i>Betreuer:</i>	Dr. A. Zisserman (University of Oxford)
<i>Ausgabedatum:</i>	01.08.1993
<i>Abgabedatum:</i>	31.01.1993
<i>Bearbeitungszeit:</i>	6 Monate

.....  
Prof. Dr.-Ing. H. Burkhardt

This thesis is entirely my own work and, except where otherwise stated, describes my own research.

Oxford, February 7, 2001

# Contents

0.1	Introduction . . . . .	1
0.2	Specification . . . . .	1
0.3	Relevant Literature . . . . .	2
<b>1</b>	<b>Introduction</b>	<b>7</b>
1.1	The Object Class of Interest . . . . .	8
1.2	The Task . . . . .	8
1.3	The Chosen Imaging Geometry . . . . .	11
1.4	Contributions of this Thesis . . . . .	12
1.5	Outline of this Thesis . . . . .	13
<b>2</b>	<b>Distinguished features</b>	<b>14</b>
2.1	Tangents . . . . .	16
2.1.1	The Tangent Cone . . . . .	16
2.1.2	The Outline . . . . .	16
2.2	The Affine Basis . . . . .	19
<b>3</b>	<b>The Weak Perspective Camera</b>	<b>21</b>
3.1	The underlying Geometry . . . . .	22
3.1.1	The Surface of Revolution . . . . .	22
3.1.2	The Weak Perspective Camera . . . . .	23
3.1.3	Recovering the Generating Function . . . . .	26

3.1.4	How to calculate the viewing direction . . . . .	28
3.1.5	Transfer using two arbitrary views . . . . .	28
3.2	Method 1. Using the Generating Curve . . . . .	30
3.2.1	Summary . . . . .	30
3.2.2	The Implementation . . . . .	31
3.2.3	Results . . . . .	35
3.3	Method 2. Using the Outline's Envelope . . . . .	37
3.3.1	The underlying Geometry . . . . .	37
3.3.2	The Implementation . . . . .	39
3.3.3	Results . . . . .	41
3.4	Comparing the two Methods . . . . .	45
3.5	Affine Extensions . . . . .	46
3.5.1	Unknown Aspect Ratio . . . . .	46
3.5.2	Full Affine Distortions . . . . .	47
<b>4</b>	<b>The Affine Camera</b>	<b>49</b>
4.1	Theoretical Background . . . . .	50
4.1.1	The Affine Camera . . . . .	50
4.1.2	The Surface's 3D Geometry and its Image . . . . .	51
4.1.3	Acquisition — Calculating the Conics . . . . .	52
4.1.4	Transfer . . . . .	53
4.1.5	Summary . . . . .	56
4.2	Implementation . . . . .	58
4.2.1	The Common Frame . . . . .	58
4.2.2	The Acquisition . . . . .	59
4.2.3	Transfer . . . . .	62
4.3	Results . . . . .	63



4.4	Possible Enhancements and Open Questions . . . . .	65
4.4.1	Better Features than Intersections . . . . .	65
4.4.2	Unused Constraints . . . . .	66
<b>5</b>	<b>The Projective Camera</b>	<b>68</b>
5.1	The underlying geometry . . . . .	69
5.1.1	The projective Camera . . . . .	69
5.1.2	The Surface's 3D Geometry . . . . .	70
5.1.3	Summary . . . . .	75
5.2	A possible Implementation . . . . .	75
5.2.1	Acquisition . . . . .	76
5.2.2	Transfer . . . . .	77
5.2.3	Transfer into the Canonical Frame . . . . .	78
5.3	Results . . . . .	78
<b>6</b>	<b>Conclusions</b>	<b>81</b>
6.1	A Recognition System . . . . .	82
6.1.1	Transfer between two Views . . . . .	82
6.1.2	Transfer into a Canonical Frame . . . . .	82
6.1.3	How to build a Recognition System . . . . .	83
6.2	Future Work . . . . .	83

# Chapter 1

## Introduction

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“Begin at the beginning,” the King said, gravely, “and go on till you come to the end: then stop.”

Lewis Carroll, *Alice in Wonderland*

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